Landau-Ginsberg Theory of Quark Confinement

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We describe the SU(3) deconfinement transition using Landau-Ginsberg theory. Drawing on perturbation theory and symmetry principles, we construct the free energy as a function of temperature and the Polyakov loop. Once the two adjustable parameters of the model are fixed, the pressure p, energy ε and Polyakov loop expectation value P_F are calculable functions of temperature. An excellent fit to the continuum extrapolation of lattice thermodynamics data can be achieved. In an extended form of the model, the glueball potential is responsible for breaking scale invariance at low temperatures. Three parameters are required, but the glueball mass and the gluon condensate are calculable functions of temperature, along with p, ε and P_F.

1. Theory

We take the free energy density f of the gluon plasma to be a function of the temperature T and the fundamental representation Polyakov loop P. The theory also depends on a renormalization group invariant scale-setting parameter Λ. Perturbation theory gives a free energy f of the form $T^4 f_4(P, g(T/\Lambda))$. Perturbation theory does not describe f near the deconfining transition, but is probably adequate for T much greater than the deconfinement temperature T_d[1][2]. Subleading terms, of the form $T^{4-r}\Lambda^r f_r(P, g(T/\Lambda))$, are likely needed to describe the deconfining transition. Such terms are inherently non-perturbative, due to the appearance of the factor Λ^r. It is easy to show that $\Delta \equiv \varepsilon - 3p$ is given by

$$\Delta = [4 - T\partial_T]\ f$$

and therefore contains information about the subleading terms. Note that Δ is also directly related to the finite temperature contribution to the stress-energy tensor anomaly, which depends in a non-trivial way on the Polyakov loop[3].

Given the close connection between Δ and the subleading terms in f which drive the deconfinement transition, it is natural to examine the behavior of $\Delta(T)$ near T_d as measured in simulations. Using the data of Boyd et al for $SU(3)$ lattice gauge theory[4], we find that $\Delta(T) \propto T^2$ over a large range of temperatures above T_d. This suggests that a term in f proportional to T^2 plays an important role in the deconfinement transition. It is also necessary to have a term proportional to T^0 (independent of T), so that there is a non-zero free energy density difference between confined and deconfined phases at very low temperatures. Thus we conjecture the simple form for the free energy

$$f(T,P) = T^4 f_4(P) + T^2\Lambda^2 f_2(P) + \Lambda^4 f_0(P)$$

where f_0 must favor the confined phase to yield confinement at arbitrarily low temperatures.

We look at the one loop perturbative result for guidance on the possible forms for f_r. We define the eigenvalues q_j by diagonalizing the fundamental representation Polyakov loop P: $P_{jk} = exp\,[i\pi q_j]\,\delta_{jk}$. The free energy for gluons in a constant A_0 background is [5][6]:

$$\begin{aligned} f_g(q) &= \frac{2}{\beta} Tr_A \int \frac{d^3k}{(2\pi)^3} \ln\left[1 - e^{-\beta\omega_{\mathbf{k}}} P\right] \\ &= -\frac{2}{\beta} Tr_A \int \frac{d^3k}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{1}{n} e^{-n\beta\omega_{\mathbf{k}}} P^n \\ &= \frac{2\pi^2 T^4}{3} \sum_{j,k=1}^{N} (1 - \frac{1}{N}\delta_{jk}) B_4\left(\frac{|\Delta q_{jk}|_2}{2}\right) \end{aligned}$$

where $|\Delta q_{jk}|_2 \equiv (q_j - q_k)\,mod(2)$ and B_4 is the fourth Bernoulli polynomial, given by $B_4(x) =$

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$x^4 - 2x^3 + x^2 - \frac{1}{30}$. The free energy is a sum of terms, each of which represents field configurations in which a net number of n gluons go around space-time in the Euclidean time direction.

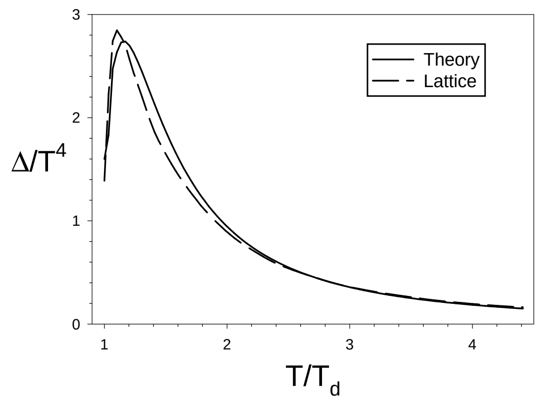

Figure 1. Δ/T^4 versus T.

2. Simple Model

In imitation of perturbation theory, we use the Bernoulli polynomial to construct f_0, f_2 and f_4 as polynomials in the q variables with the appropriate symmetries. There are two inequivalent directions in the Cartan subalgebra of $SU(3)$, λ_3 and λ_8. Confinement is achieved by motion in the λ_3 direction away from $A_0 = 0$. Defining $P_F = Tr_F(P)$, we parametrize motion along the line $Im P_F = 0$ by

$$P_F(\psi) = 1 + 2\cos\left[2\pi(1-\psi)/3\right]$$

with $\psi = 0$ giving $P_F = 0$, and $\psi = 1$ corresponding to $A_0 = 0$ and $P_F = 3$. We take the free energy density to have the form

$$f(\psi, T) = aT^4\left(\psi^4 - \frac{2}{3}\psi^3 + \psi^2\right) + \left(b + cT^2\right)\psi^2$$

where $a = 4\pi^2/15$; b and c fix the critical properties. This potential can be extended to the entire Lie algebra, and contains all required symmetries. For low temperatures, the $b\psi^2$ term dominates. If $b > 0$, the system will be confined. The parameter b can be interpreted as the free energy difference at $T = 0$ between the $\psi = 0$ confined phase and the fully deconfined $\psi = 1$ phase.

3. Results

The above potential has built in the correct low and high temperature behavior, and has two free parameters, b and c. We can use one of these to set the overall scale by fixing the deconfinement temperature. To determine the remaining parameter, we fit the lattice data for Δ at $N_t = 8$, which is well measured and a good approximation to the continuum limit[4]. With $T_d = 0.272\,GeV$, we obtain $b^{1/4} = 0.356\,GeV$ and $c^{1/2} = 0.313\,GeV$. The results of our fitting procedure are shown in figures 1, 2 and 3 for Δ, p and ε. The agreement is good throughout the range $T_d - 4T_d$. The discrepancy in the high-temperature behavior of p and ε is probably accounted for by HTL-improved perturbation theory[1][2].

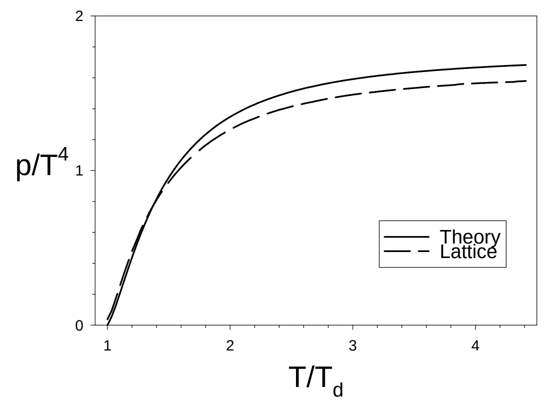

Figure 2. p/T^4 versus T.

4. Extended Model

The physical origin of the parameters b and c above is obscure. Since the trace of the stress-energy tensor θ^μ_μ couples to the scalar glueball, we introduce a scalar glueball field ϕ as the source of scale symmetry breaking in an extended model. For $SU(3)$, our extended model is

$$\begin{aligned} f &= aT^4\left(\psi^4 - \frac{2}{3} + \psi^2\right) + \left(\alpha\phi^4 + \beta\phi^2 T^2\right)\psi^2 \\ &\quad + \lambda\phi^4 \log\left(\frac{\phi^2}{e^{1/2}\mu^2}\right). \end{aligned}$$

Spontaneous symmetry breaking of ϕ via a Coleman-Weinberg potential introduces the scale

μ. If we make the identification $\phi^4 \propto Tr\,\left(F_{\mu\nu}^2\right)$, the $T = 0$ potential for ϕ can be derived in a variety of ways: 1) renormalization group [7]; 2) explicit calculation for constant fields[8]; 3) stress-energy tensor anomaly [9]; 4) stress-energy sum rules [10]. The values of λ and μ can be determined from the values of the gluon condensate and the glueball mass. The $T = 0$ condensate from the Coleman-Weinberg potential is given by $-2\lambda\phi^4 \rightarrow -2\lambda\mu^4$and the glueball mass is given by $M_s^2 = 8\lambda\mu^2$. A similar glueball potential has been used to model the chiral transition [11]. A coupling between ϕ and P can be inferred from perturbation theory [3] [12]; similar couplings to the chiral order parameter exist [13] [14][15].

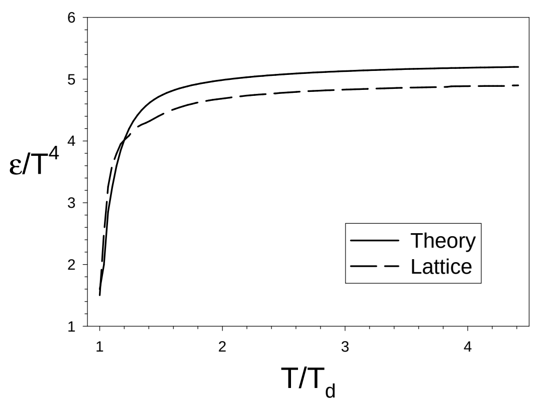

Figure 3. ε/T^4 versus T.

We have found values for the parameters α, β, λ and μ which mimic the behavior of our simpler model near T_d. Our extended model has a potentially fatal problem associated with the restoration of scale symmetry. For plausible values of the gluon condensate and glueball mass, restoration of scale symmetry at T_d leads to a single abrupt phase transition incompatible with lattice data. The alternative, with unrealistic values, is restoration above T_d via a first order transition, which would be observable in lattice data. This argues against any simple role of the glueball in the thermodynamics of the gluon plasma.

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